Four Corner Magic Squares 6×6 with semi Symmetric Center 2×2

Muftah ,.A ,.ELHAMALE

Mathematics, Department, science college, Azzaytuna University.

Abstract:

In this paper, we consider one type of the classical magic squares 6×6 having the four corner property, namely the Magic Squares with 2×2 Semi Symmetric Center, We list counting of this type and some of their algebraic properties. We present the general form of this type. We classified these sets of squares according to their eigenvalues and determinant, We give an example to each class in our classification, In each example we give the basis of the nullspase of the square as well as the eigenvalues and the determinant of this square.

Our calculations and proofs are based on applying the theorems and methods of a linear algebra and computer programs, specially designed for matrix calculations

Keywords: Magic Squares, four corner, Semi symmetric Center, matrix, eigenvalues

الملخص:

يرجع تاريخ المربعات السحرية إلى مئات السنين قبل الميلاد وإلى الصين بالتحديد، وكذلك إلى بابل والهند، وكانت أول مطالعة علمية أجريت على المربعات السحرية عام 1420م من قبل قس يوناني، ثم طالع علماء مسلمين هذه المربعات، كذلك علماء رياضيات مشاهير مثل أويلر وفرما، وحديثاً توسعت الدراسة لتشمل طرق حساب وتركيب وتوليد المربعات، وخواصها وإرتباطها بعلاقات مع

المصفوفات والجبر الخطي واستخدمت لدراستها لغات البرمجة وبرمجيات متقدمة، وقد تركزت الدراسة على مربعات سحرية غير تقليدية تتسم بصفات إضافية، ومنها المربعات السحرية شبه متماثلة المركز 2×2 ذات خاصية الزوايا الأربعة، وقد قمنا في هذا البحث بتعداد المربعات السحرية من النوع 6×6 شبه متماثلة المركز 2×2 ذات خاصية الزوايا الأربعة، وقد صنفنا التعداد إلى عدة مجموعات حسب نوع المركز شبه المتماثل 2×2. وكذلك قمنا بصياغة الشكل العام للمربعات السحرية من النوع 6×6 شبه متماثلة المركز 2×2 ذات خاصية الزوايا الأربعة. ثم درسنا بعض خواصها الجبرية من خلال قيمها الذاتية، معتمدين في الإثبات والحساب على طرق الجبر الخطي وبرامج حاسوب تختص بذلك.

Introduction:

The magic square is a square matrix, where the sum of all entries in each row or column and both main diagonals yields the same number[1]. This number is called the magic constant, A natural magic square of order n is a matrix of size n×n such that its entries consists of all integeres from one to n^2. The magic constant is in this case $S=\frac{n(n^2+1)}{2}$;[8].

A pandiagonal magic square is a magic square such that the sum of all entries in all bent-diagonals equals the magic constant. A symmetric magic square is a natural magic square of order n such that the sum of all opposite entries equals n + 1 ;[6].

The number of natural magic squares of order 6 is til now unknown ;[3]. We give here the number of a subset of such squares. It is well-known that there are no pandiagonal magic squares nor Semi Symmetric squares of order 6. We dene here classes of magic squares of order 6, which satisfy some of the conditions for both types.

Material and methods

Defnition 1: A four corner magic square of order 6 is magic square $\lfloor a_{ij} \rfloor$, $i = 1,...,6$, $j = 1,...,6$ with magic constant $3s$ such that $a_{ij} + a_{(i+3)(j+3)} + a_{i(j+3)} + a_{(i+3)j} = 2s$ holds for each $i = 1,2,3$, $j = 1,2,3$ and

$$a_{33} + a_{44} + a_{34} + a_{43} = 2s$$

The entries of a four corner magic square of order 6 satisfy:

$$a_{14} + a_{25} + a_{36} + a_{41} + a_{52} + a_{63} = 3s \quad ,$$
$$a_{14} + a_{25} + a_{36} + a_{41} + a_{52} + a_{63} = 3s$$

These two conditions represent the sum of the entries of two broken diagonals. If the magic square is pandiagonal, then we have to consider all broken diagonals. To see the validity of the .rst equation we know from the definition that.

$a_{11} + a_{44} + a_{14} + a_{41} = 2s$, $a_{22} + a_{55} + a_{25} + a_{52} = 2s$, $a_{33} + a_{66} + a_{36} + a_{63} = 2s$

holds Adding up these equations and subtracting from them the following equation: $a_{11} + a_{22} + a_{33} + a_{44} + a_{55} + a_{66} = 3s$

yields the desired equation.

Defnition 2: A four corner magic squares with Semi Symmetric center is a four corner magic square of order 6 such that:

$a_{33} + a_{44} = s$, $a_{34} + a_{43} = s$

This means that the 2 by 2 square in the center is semi Symmetric. A four corner magic squares with Symmetric center is a four corner magic square of order 6 can be written as:

X	f	G	T	G	M	(1)
Z	h	N	J	Q	3s − j − n − q − h− z	
W	H	E	A	M	J	
$2s - b - t - x$	k	$2s - b - a - e$	B	D	R	
2s-o-j-z	p	D	O	$2s - h - p - q$	T	
B	L	A	$3s - b - j - o - a - t$	E	F	

where

$$A = a + b - d - g - n + s,$$
$$B = b + j + o - s + t - w,$$
$$D = g - j - k - o - p - q + s + w + x + e,$$
$$E = f + h + k - m + p - s,$$
$$F = p - b + q + s - x - e,$$
$$G = j - g - f + o + p + q + s - w - x - e,$$
$$H = b + j + o - s + t - w,$$
$$J = d - a + g + n - p - q + x,$$
$$L = b + j + o - s + t - w,$$
$$M = 2s - o - p - q - j - t + w + e,$$
$$R = a + b - g + j + o + p + q - 2s + t - w,$$
$$T = h - d + j + q - s + z.$$

We note that preious form is a magic square of type 6×6 with the property of four corner represented by following equations:

$$a_{13} + a_{22} + a_{31} + a_{61} + a_{55} + a_{64} = 3s,$$
$$a_{14} + a_{25} + a_{36} + a_{41} + a_{52} + a_{63} = 3s,$$
$$a_{33} + a_{44} + a_{34} + a_{43} = 2s.$$

we will denote this type by symbol mfcsi, magic square 6×6 with the property of four corner with the semi Symmetric Center, and the form is in the following theorem:

Theorem3: The general form of a magic square 6×6 with the property of four corner and semi Symmetric Center 2×2, is given in the following form, and the conditions for solving counting problem.

Proof: From the defintion of the square mentioned, we have the following linear system of equations for the square case $[a_{ij}]$, where $i, j = 1, \ldots, 6$

with magic costant which consists of 25 equations, and 36 variables, such that for every a_{ij} then**:**

$a_{ij} + a_{(i+3)(j+3)} + a_{i(j+3)} + a_{(i+3)j} = 2s$ where $i \leq 3,\ j \leq 3,$

By solving this system, we have a magic square in the following form:

X	f	G	T	G	M	
Z	h	N	J	Q	$3s - j - n - q - h - z$	(2)
W	H	E	A	M	J	
$s + a - t - x$	k	$s - e$	$s - a$	D	R	
$2s - o - j - z$	p	D	O	$2s - h - p - q$	T	
B	L	$2s - g - n - d$	$2s - j - o - t$	E	F	

Where:

$B = b + j + o - s + t - w,$
$D = g - j - k - o - p - q + s + w + x + e,$
$E = f + h + k - m + p - s,$
$F = p - b + q + s - x - e,$
$G = j - g - f + o + p + q + s - w - x - e,$
$H = b + j + o - s + t - w,$
$J = d - a + g + n - p - q + x,$
$L = b + j + o - s + t - w,$
$M = 2s - o - p - q - j - t + w + e,$
$R = a + b - g + j + o + p + q - 2s + t - w,$
$T = h - d + j + q - s + z.$

We see that it has seventeen independent variables, We can consider a special class of the class of four corner magic squares with Semi Symmetric center, also to the following two replacement the cells

a_{25} (res. a_{52}) with a_{55} (res. a_{22}) , a_{44} (res. a_{24}) with a_{53} (res. a_{23})

The squares, which can be written in the following form:

X	F	G	T	G	M	
Z	$2s - h - p - q$	D	O	P	$3s - j - n - q - h - z$	
W	H	E	A	M	J	(3)
$s + a - t - x$	K	$s - e$	$s - a$	D	R	
$2s - o - j - z$	Q	N	J	H	T	
B	L	$2s - g - n - d$	$2s - j - o - t$	E	F	

In order to eliminate the effect of the previous Replacement for magic squares of type 6×6 with the property of four corners centered at 2×2 semi-symmetric, implement the following conditions. $1 \le a \le 17$, $a < e < s - a$, $p < q$ ..(1). Therefore, we will multiply the total number of this type of natural squares by 16 to add to the previous Replacement. You can transform the general shape of this square according to Condition 1 into the following form:

X	f	G	T	G	M	
Z	$p+q+h-s$	N	J	$s-p$	$3s-j-n-q-h-z$	...(4)
W	H	E	A	M	J	
$s+a-t-x$	k	$s-e$	$s-a$	D	R	
$2s-o-j-z$	$s-q$	D	O	$s-h$	T	
B	L	$2s-g-n-d$	$2s-j-o-t$	E	F	

We notice that the previous forms maintain the type and that the center 2×2 has not changed in all cases. Also, the condition that $a_{52} < a_{25}$ is satisfied in all cases. Therefore, these are natural magic squares with the property of four corners and a semi-symmetric center, and the constants e and a will be even.

Result 1: Count of Squares mfcsi:

We used the computer and designed a special program to solve this problem, which includes defining a 6x6 magic square with the property of having four corners and a semi symmetric center of 2x2. We linked this type to a number of files, each containing a different value, as shown in the following table:

a =1							
e =	Number	e =	Number	e =	Number	e =	Number
2	116511842	11	295406900	20	332716362	28	358076270
3	132987568	12	325569546	21	332269470	29	350383960
4	201772222	13	314724780	22	334768630	30	360519204
5	195213542	14	330356970	23	343668656	31	351628294
6	247269840	15	319871530	24	353535258	32	352098982
7	249849504	16	338570808	25	351413848	33	347005202
8	279043734	17	319583422	26	354646898	34	346545330
9	282858168	18	334466160	27	356452980	35	336033008
10	314225850	19	334597154				
a =2							
e =	Number	e =	Number	e =	Number	e =	Number
3	206302610	11	331287390	19	326515794	27	356484732
4	210992494	12	321640070	20	345399018	28	361179738
5	251163058	13	350613710	21	335036204	29	374039976
6	245105736	14	332114584	22	333026268	30	344502648
7	274508468	15	336104578	23	356959948	31	353814694
8	287150618	16	331537886	24	360229516	32	363592188

9	296599162	17	350953660	25	353925968	33	362587560
10	302170466	18	311139312	26	358846606	34	347566238
a =3							
e =	Number	e =	Number	e =	Number	e =	Number
4	251739982	12	336328382	20	336807852	27	360898736
5	240505110	13	324869294	21	336809760	28	363278990
6	273691912	14	329469256	22	339051514	29	346910646
7	275204588	15	324711266	23	344044096	30	366423556
8	299899266	16	337436298	24	355226184	31	352598114
9	294890678	17	328147564	25	362975754	32	354459440
10	308909490	18	323876138	26	353891828	33	361775374
11	306028538	19	320541794				
a =4							
e =	Number	e =	Number	e =	Number	e =	Number
5	277262012	12	329152950	19	335007752	26	353888706
6	278106484	13	341948432	20	326708492	27	366826140
7	305137672	14	327143618	21	341404426	28	354933908
8	286925428	15	341003048	22	346408234	29	372809534
9	318200762	16	329377468	23	354547734	30	354855806
10	326734318	17	322104494	24	358998042	31	365478056
11	325879172	18	323967196	25	358784510	32	354459440
a =5							
e =	Number	e =	Number	e =	Number	e =	Number
6	291627212	13	335041250	20	346739100	26	381023514

7	289665812	14	386116304	21	329511150	27	355241148
8	319609974	15	332832472	22	343736302	28	353360938
9	303484650	16	341768070	23	373008254	29	362792908
10	319630542	17	320454620	24	358755138	30	367872592
11	331302656	18	328627682	25	368793648	31	356279070
12	329152040	19	323827718				
a =6							
e =	Number	e =	Number	e =	Number	e =	Number
7	307831520	13	335408322	19	327838034	25	362278616
8	307220566	14	323632092	20	325045442	26	344707584
9	326632592	15	343900408	21	346521984	27	357310716
10	315214080	16	317359492	22	346273390	28	352453064
11	329922646	17	329088296	23	366929874	29	364671418
12	337240842	18	324929256	24	357798606	30	354408080
a =7							
e =	Number	e =	Number	e =	Number	e =	Number
8	318913908	14	338270640	20	343814008	25	353600518
9	313804886	15	321017534	21	338858100	26	353855988
10	332965950	16	331596514	22	361348270	27	352947394
11	312416444	17	315920958	23	344125594	28	369485474
12	339820720	18	333891060	24	350598284	29	361325404
13	326680272	19	334337068				
a =8							
e =	Number	e =	Number	e =	Number	e =	Number

9	322651726	14	336499586	19	326604342	24	344790314
10	313931550	15	327023778	20	351929926	25	345471054
11	350259448	16	321879182	21	342614872	26	371263102
12	328728052	17	349000474	22	338372718	27	368869600
13	331352496	18	332648602	23	355184854	28	358386792
a =9							
e =	Number	e =	Number	e =	Number	e =	Number
10	332885406	15	312280764	20	335943324	24	359800942
11	325241400	16	327204636	21	326267006	25	363693102
12	342083312	17	322272062	22	338824226	26	353751236
13	321193778	18	331971266	23	342484208	27	369395860
14	336616920	19	340019774				
a =10							
e =	Number	e =	Number	e =	Number	e =	Number
11	346174266	15	320420772	19	333407464	23	351659296
12	331299312	16	314295184	20	317775590	24	348631644
13	343459744	17	329404430	21	334963576	25	360576518
14	316652498	18	315451068	22	324426154	26	346157032
a =11							
e =	Number	e =	Number	e =	Number	e =	Number
12	319612194	16	327835916	20	330765638	23	345843756
13	311520480	17	318252788	21	355567308	24	359195928
14	334495182	18	318965356	22	339579470	25	368711314
15	7807314	19	315707450				

a =12							
e =	Number	e =	Number	e =	Number	e =	Number
13	333424860	16	310667578	19	330888746	22	335530678
14	307147362	17	328751622	20	321671738	23	355516716
15	315379026	18	315313592	21	353451262	24	346475818
a =13							
e =	Number	e =	Number	e =	Number	e =	Number
14	323015258	17	312747912	20	341748002	22	353459532
15	308092470	18	340473770	21	331096842	23	355448444
16	326470472	19	331125814				
a =14							
e =	Number	e =	Number	e =	Number	e =	Number
15	315555354	17	372360568	19	334305526	21	347812602
16	313515758	18	322125932	20	370518284	22	326982930
a =15							
e =	Number	e =	Number	e =	Number	e =	Number
16	342594820	18	351825242	20	341764226	21	340531500
17	321828952	19	318003462				
a =16							
e =	Number	e =	Number	e =	Number	e =	Number
17	339798226	18	342610906	19	324696620	20	328137776
a =17							

e =	Number	e =	Number				
18	398369256	19	344164516				

The number of squares of this type is:

50, 712 , 530, 499×2 =101, 425, 060, 998

The total number of squares is:

101425060998 × 16 = 1,622,800,975,968

Result 2: following classification according to the form of the characteristic Polynomial .

Type	Characteristic polynomial
I	$(x-111)(x^5+hx^4+fx^2+gx+k)$
II	$x(x-111)(x^4+fx^2+gx+k)$
III	$x^2(x-111)(x^3+hx+k)$

where. $k \neq 0$

According to the structure of the eigenvalues we have the following classification:

type	Eigenvalues
A	$x_i = 111, a, b, c \ , y+iz, y-iz$
B	$x_i = 111, a \ , y+iz, y-iz \ , y+iw, y-iw$
C	$x_i = 111, a, b, c, d, e$

According to the structure of the absolute value of the eigenvalues we have the following classification:

Type	Absolute value of the eigenvalues
1	$x_i = a, b, c, d, e, 111$
2	$x_i = a, a, b, c, d, 111$
3	$x_i = a, a, b, b, c, 111$
4	$x_i = a, a, a, b, c, 111$
5	$x_i = a, a, a, a, b, 111$

where: $0 < a < b < c < d < e < 111$ and $i = 1, \ldots, 6$.

Examples:

-1

9	17	22	10	32	21
7	4	34	35	1	30
19	23	13	12	26	18
31	11	25	24	14	6
29	36	2	3	33	8
16	20	15	27	5	28

nullspace basis =

-1
0
-1
2
1
-1

$p(x) = x^2 \left(x^4 - 111x^3 + 23x^2 + 710x - 107594 \right)$

$x_i = 0, 0, 111, -20.93, 10.47 + 18.75i, 10.47 - 18.75i$

-2

25	29	15	7	18	17
27	3	21	26	2	32
28	23	1	4	24	31
6	13	33	36	14	9
5	35	11	16	34	10
20	8	30	22	9	12

nullspace basis=

-1
0
1
-1
0
1

$$p(x) = x(x-111)(x^4 - 215x^2 - 3366x - 31296)$$

$$x_i = 111, 0, 21.09, -12.21, -4.44+10.09i, -4.44-10.09i$$

-3

17	12	23	18	31	10
26	2	24	28	1	30
29	21	3	4	22	32
5	15	33	34	16	8
7	36	9	13	35	11
27	25	19	14	6	20

det = - 47072880

$$p(x) = x^6 - 111x^5 + 29x^4 + 1037x^3 - 476724x^2 + 902268x - 47072880$$

$$x_i = 111, 17.52, 9.03+12.91i, 9.03-12.91i, -0.27+9.87i, -0.27-9.87i$$

- 4

9	20	23	19	29	11
31	3	21	22	2	32
30	24	1	4	25	27
10	12	33	36	13	7

det = 17118864

5	35	15	16	34	6
26	17	18	14	8	28

$$p(x) = (x - 111)(x^5 - 163x^3 - 18x^2 - 20808x - 154224)$$

$$x_i = 111, 16.55, -14.5, -6.08, 2.01 + 10.08i, 2.01 - 10.08i$$

Result 2: Estimating the number of squares:

We want to estimate the number of magic squares with four corner property of type 6×6, by calculating the number of squares at centar 2×2, where we found the number of these squares with rhe possiblr and different values of a , e and we have:

$$2 + 4 + 6 \ldots + 34 = 306$$

The average of square of all possible values of a, e

$$\frac{101425060998 \times 16}{306} = 5{,}3033 \times 10^9$$

The number of all possible and different values of a, b, e, was calculated using the computer through the magic squares with four corner property, which is 3429.

Therefore, we guess the number of nutural magic squares with four corner property of type 6×6.

$$5{,}3033 \times 10^9 \times 3429 = 1{,}8185 \times 10^{13}$$

References

[1] Ahmed, Maya, **Algebraic Combinatorics of Magic Squares**, Ph.D. Thesis, Mathematics Dep., University of California, Davis, (2004).

[2] Al-Zahawi, A., & Al-Ashhab, S., **Linear Systems Resulting From Pandiagonal Magic Squares**, Accepted for Publication at ALmanara Journal, AL-Albayt University (2006).

[3] Anton, H., **Elementary Linear Algebra**, 7th Edition, Jon Wiley, New York, (1994).

[4] Ashhab S., 1998 - **Computer Solutions of The Magic (4×4) Squares and Semi Magic (4×4) Squares Problem**, *Dirasat Natural and Engineering Science*, **25**, 445-450.

[5] Ashhab S., 1999 - **The Middle Number In The Magic (5×5) Squares**, *Dirasat. Pure Sciences*, **26**, 248 -254.

[6] Ashhab S., 2006 -**Non-Consecutive Magic Squares (4×4)**, The Conference of Gaza Islamic Univ.

[7] Ashhab S., 2008- **Magic Squares 5×5,** International Journal of Applied Science and Computations.

[8] Barton, C. & Sudbery, A., **Magic Squares and Matrix Models** of Lie Algebras, University of York (2002).

[9] Christopher , J ., Henrich , **Magic Squares and Linear Algebra** , Amer Math , Monthly ,Vol , 98 ,No , 6 , Jun ., 1991 , pp . (481-488) .

[10] Essen A., 1990 - **Magic squares and linear algebra**, *American Mathematical Monthly*, **97**, 60-62.

[11] Gauthier N., 1997- **Singular matrices applied to 3×3 magic squares**, *The Mathematical Gazette*, **81**, 225-230.

[12] Hendricks, J., **The Determinant of a Pandiagonal Magic Square of Order4** Is Zero, Journal of Recreational Mathematics, Vol. (21), (1989), p.p. (179-181).

[13] Loly P., 2003- A **purely pandiagonal 4×4 square and the Myers Briggs type Table**, *J. Rec. Math*., **31(1),** 29-31.

[14] Loly P., 2004- **The Invariance of the Moment of Inertia of Magic Squares**, *The Mathematical Gazette*, **88**, 151-513.

[15] Loly P.; Trump W.; Schindel D., 2009- **The eigenvalues of low order singular and non-singular magic squares** 00A08, 05B20, 11C20, 15A03, 15A18.

[16] Loly, P. D. & Steeds, M. J., A **New Class Of Pandiagonal Squares** University of Manitoba, July 6, 2004.

[17] Lee, M., & etal, **Linear Algebra of Magic Squares**, (2006) www.cst.cmich.edu/ users/smith1kw/REU 2006 lee love washer. Pdf.

[18] Palm.; William J., 2005- **Introduction to MATLAB 7 For engineers**, Mc Graw-Hill, Boston.

[19] Yee Tony, **Applied Linear Algebra & Differential Equations**, The Hong Kong University Of Science and Technology, 2006